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Camouflaged supersymmetry in solutions of extended supergravities

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We establish a relation between certain classes of flux compactifications and certain families of black hole microstate solutions. This connection reveals a rather unexpected result: there exist supersymmetric solutions of $N = 8$ supergravity that live inside many $N = 2$ truncations, but are not supersymmetric inside any of them. If this phenomenon is generic, it indicates the possible existence of much larger families of supersymmetric black rings and black hole microstates than previously thought.

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There is an extensive body of work on obtaining supersymmetric and nonsupersymmetric vacua for flux compactifications of string theory and studying their phenomenology, and a parallel extensive body of work on constructing supersymmetric and nonsupersymmetric black hole microstate solutions to understand black hole physics in string theory. While the physical motivations are different, the technical tools are rather similar. In particular, the equations underlying supersymmetric solutions are well-understood and classified; on the flux compactification side (see for example Refs.[1–3]) in ten dimensions, and on the black hole microstate side for the underlying supergravity in five dimensions [4–6]. Furthermore, some of the methods for constructing nonsupersymmetric solutions from supersymmetric ones are strikingly similar. These methods include slightly deforming the supersymmetric solution by additional fluxes [1,3], flipping some signs [7], or writing some effective Lagrangian as a sum of squares for black holes [8–14] or flux backgrounds [15,16].

It is therefore not surprising that one can find a relation between certain types of solutions on the two sides. Indeed, as we will show below, certain supersymmetric flux backgrounds of the type in Ref. [17] where the "internal" (noncompact) manifold contains a hyperkähler factor can be interpreted as certain nonrotating solutions in the classification of Refs. [4–6]. (One can similarly relate nonsupersymmetric solutions. The story is more intriguing and is alluded to in this letter, but we leave the details, and an explicit solution, for a companion publication [18].) The main purpose of this letter is to show that there are other supersymmetric solutions of the same class of flux compactifications which, when interpreted as black hole microstates in $N = 2$ supergravity, do *not* fall into the classification of supersymmetric solutions [4–6]. (We use four-dimensional supersymmetry conventions. For instance, all $N = 2$ theories, regardless of dimension, have eight supercharges.) Hence, from the point of view of $N = 2$ supergravity, these solutions should be nonsupersymmetric. However, they are supersymmetric inside $N = 8$ supergravity!

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As we will explain below, these solutions have the right field content to fit into many possible $N = 2$ truncations, and hence they will always be solutions of these $N = 2$ theories. However, the unbroken supercharges are projected out in all possible $N = 2$ truncations and hence from the point of view of $N = 2$ supergravity none of these solutions are supersymmetric. A simple way to understand this is to recall that all $N = 2$ supersymmetric solutions in the class [4–6] have (in our conventions) anti-self-dual fields on a hyperkähler base, while our solutions have *both* anti-self-dual and self-dual fields.

The fact that a nonsupersymmetric solution of an $N = 2$ or an $N = 4$ theory can become supersymmetric when embedded in $N = 8$ has been know for quite a while. In particular, a large $N = 8$ BPS black hole is either BPS or nonBPS in an $N = 2$ truncation, depending on whether the Kähler covariant derivative of the central charge vanishes or not [19–21]. However, our solutions do not fall into these classes. They can have multiple centers on the four-dimensional hyperkähler space and therefore may depend on four coordinates. If we restrict to the subset of solutions with a single center, we only find small black holes, since we have only one electric charge (and four types of dipole charges) [22]. In order to find four-dimensional single center solutions, we can choose the hyperkähler space to be Taub-NUT. In this case, the quartic invariant of the charges vanishes so that the black hole will always have only a small horizon. For multicenter solutions, though, the story is more complicated.

Our results have quite a few unexpected implications. First, it is widely believed that all supersymmetric microstate geometries of three-charge black holes in five dimensions are described by the equations of Refs. [4–6]. Our results indicate that many solutions that are not described by these equations are also supersymmetric in the parent $N = 8$ theory. This implies that besides the classes of microstate solutions constructed so far there may exist many more supersymmetric microstates, which would contribute to the entropy count.

Second, it has been conjectured [23] and argued that all multicenter supersymmetric solutions of $N = 8$ supergravity must live inside an $N = 2$ truncation [24] or structure [25], and one may believe that this implies that the solutions of Refs. [11,26] capture all supersymmetric multicenter $N = 8$ solutions. Our results show that this is not so.

Third, it is well-known that the supersymmetric black ring in five dimensions [4,27–29] is part of a truncation to $N = 2$ ungauged supergravity and belongs to the class of solutions of Refs. [4–6]. Our results indicate that there may exist a new, more general supersymmetric black ring with more dipole charges (coming from the extra self-dual fluxes). Besides its interest as a new solution, if this black ring existed, it may also help to account for the missing entropy between the D1-D5 CFT and the dual bulk in the moulting black hole phase [30].

In general, the relation between black hole microstates and flux compactifications that we outline will likely prove fruitful in both directions. There exists a whole methodology for constructing flux compactifications by writing the effective Lagrangian governing these compactifications as a sum of squares of calibrations [15,16]. Under the guise of "floating branes," calibrations have also been used to find nonsupersymmetric black hole microstates [31], and relating the two approaches is likely to yield novel classes of solutions on both sides. We plan to report on this relation in an upcoming companion paper [18]. Furthermore, it has been recently discovered that even some nonextremal cohomogeneity-two black holes, black rings and microstates are calibrated [32]. If one could use this to write down a new decomposition of the effective Lagrangian (similar to the one of nonextremal cohomogeneity-one solutions [33–36]) one would obtain a systematic method to construct new highly nontrivial and physically interesting solutions.

I. THE SOLUTION

We focus on a class of solutions to five-dimensional $N = 8$ supergravity that arises as the low-energy limit of a T^6 compactification of 11-dimensional supergravity. The spatial part of the five-dimensional spacetime is given by a hyperkähler space M_4, and the warp factor A depends only on the M_4 coordinates. The full 11-dimensional metric is

$$\begin{aligned} ds_{11}^2 = &-e^{-2A}dt^2 + e^{A}ds^2(M_4) + e^{A}(dx_5^2 + dx_6^2 + dx_7^2 + dx_8^2) \\ &+ e^{-2A}(dx_9^2 + dx_{10}^2) \end{aligned} \tag{1}$$

with coordinates $x^5 \ldots x^{10}$ on T^6. The four-form field strength is

$$\begin{aligned} F_4^{\mathrm{mag}} = &\, d(e^{-3A}) \wedge dt \wedge dx_9 \wedge dx_{10} + [\Theta_+ - \Theta_-] \wedge dx_5 \wedge dx_8 \\ &+ [\Theta_+ + \Theta_-] \wedge dx_6 \wedge dx_7 \\ &+ \tilde{\Theta}_+ \wedge (dx_6 \wedge dx_8 - dx_5 \wedge dx_7), \end{aligned} \tag{2}$$

where Θ_+, $\tilde{\Theta}_+$ are self-dual two-forms on M_4 and Θ_- is an anti-self-dual one. With hindsight, we focus on a solution whose self-dual forms obey the relation

$$(\Theta_+ + i\tilde{\Theta}_+) \wedge (\Theta_+ + i\tilde{\Theta}_+) = 0, \tag{3}$$

which implies that $\Theta_+ + i\tilde{\Theta}_+$ defines a complex structure on M_4 under which it is a holomorphic two-form. As we see below, this ensures that the solution is supersymmetric. Finally, the warp factor is determined by

$$\Delta_4 e^{3A} = (\Theta_+^2 + \tilde{\Theta}_+^2 + \Theta_-^2) + \rho_{M2}, \tag{4}$$

where Δ_4 is the Laplacian on M_4 and ρ_{M2} the M2 brane density.

This solution has the electric charge of a set of M2 branes extended along the x_9 and x_{10} directions and smeared on the other compact directions of T^6. The magnetic component of the four-form can be thought of as being sourced by four types of M5 branes on the corresponding Poincaré dual cycles. We summarize that in Table I.

We show that this solution is a supersymmetric solution of 11-dimensional supergravity. By swapping the roles of M_4 and $T^2_{9,10}$ as external and internal spaces, we see that the above solution is actually an eight-dimensional Calabi-Yau "compactification" of M-theory, of the type discussed first in Ref. [17]. Eleven-dimensional spacetime has the form $\mathcal{M}_{1,10} = \mathcal{M}_{1,2} \times X_8$, with $X_8 = M_4 \times T^4_{5,6,7,8}$. The metric and the gauge field preserve three-dimensional Poincaré invariance, as can be seen by rewriting (1) and (2) as

$$\begin{aligned} ds_{11}^2 &= e^{-2A}(-dt^2 + dx_9^2 + dx_{10}^2) + e^{A}ds^2(X_8), \\ F_4 &= d(e^{-3A}\mathrm{vol}_3) + \mathrm{Im}[(\Theta_+ - i\tilde{\Theta}_+) \wedge dz \wedge dw \\ &\quad + \Theta_- \wedge dz \wedge d\bar{w}], \end{aligned} \tag{5}$$

where $\mathrm{vol}_3 = dt \wedge dx_9 \wedge dx_{10}$ is the volume form of three-dimensional spacetime and A only depends on the coordinates of the internal manifold X_8. Furthermore, we defined the holomorphic one-forms

$$dz = dx_5 + i dx_6, \qquad dw = dx_7 + i dx_8. \tag{6}$$

The supersymmetry conditions require $ds^2(X_8)$ to be a Calabi-Yau metric for X_8 and the internal components of F_4 to be a primitive (2, 2)-form. The first requirement is fulfilled since (1) and (2) give a Calabi-Yau metric $ds^2(X_8) = ds^2(M_4) + dzd\bar{z} + dwd\bar{w}$. Since the anti-self-dual two-forms on hyperkähler manifolds are (1, 1),

TABLE I. The brane charges for our configurations along the T^6 directions $x_5 \ldots x_{10}$. A brane is localized in directions marked "×" and smeared in the other ones. The M5 branes each wrap a 1-cycle γ_i in the hyperkähler space M_4, determined by the (anti-)self-dual fields $\Theta_\pm$, $\tilde{\Theta}_+$.

	0	9	10	5	6	7	8	M_4
M2	×	×	×					
M5	×	×	×	×			×	γ_1
M5	×	×	×		×	×		γ_2
M5	×	×	×		×		×	γ_3
M5	×	×	×	×		×		γ_4

Eq. (5) implies that the internal components of F_4 indeed make up a primitive (2, 2)-form if $(\Theta_+ + i\tilde{\Theta}_+) \wedge dz \wedge dw$ is the holomorphic four-form of X_8 [such that $(\Theta_+ - i\tilde{\Theta}_+)$ is antiholomorphic on M_4]. This in turn can only be realized if condition (3) holds. The equation of motion for the gauge field then determines the warp factor in general as $d *_8 dA = \frac{1}{6} F_4^{\rm mag} \wedge F_4^{\rm mag}$, which reduces to (4) when $X_8 = M_4 \times T^4_{5,6,7,8}$. Note that this background is dual to a supersymmetric flux background of IIB string theory in the GKP class [1,3].

Finally, we can interpret our supersymmetric solution in 11-dimensional supergravity compactified on a six-torus $(T^6_{(5,6,7,8,9,10)})$ which descends to five-dimensional $N = 8$ supergravity. There exists a very large class of solutions to this theory that fit inside an $N = 2$ truncation with two vector multiplets: they describe black rings and black holes, as well as microstate solutions that have the same charges as these objects but no horizon.

All supersymmetric solutions of this truncation are known [4,6], and are given by

$$\begin{aligned} ds^2_{11} &= -Z^{-2}(dt + k)^2 + Z ds^2_4 + Z \sum_{I=1}^{3} \frac{ds^2_I}{Z_I}, \\ F_4 &= dA^{(I)} \wedge \omega_I = \sum_{I=1}^{3} \left(-d\left(dt + \frac{k}{Z_I} \right) + \Theta^{(I)} \right) \wedge \omega_I, \end{aligned} \tag{7}$$

where $Z \equiv (Z_1 Z_2 Z_3)^{1/3}$, ds^2_I and ω_I are, respectively, a unit metric and a unit volume form on the three T^2's inside T^6 and ds^2_4 is a four-dimensional hyperkähler metric. When this metric has a translational $U(1)$ isometry it becomes a Gibbons-Hawking metric: if one then compactifies along the Gibbons-Hawking fiber, one obtains a solution of the four-dimensional STU model. Note that we work in a convention in which the three curvature two-forms of the hyperkähler base are self-dual, and hence the $\Theta^{(I)}$ of a supersymmetric solution are anti-self-dual.

The metric and the timelike (electric) components of the four-form of our solution (1) and (2) are of the form (7) with $Z_1 = Z_2 = 1$ and $k = 0$. However, the spacelike (magnetic) four-form field strengths have more components, and only reduce to the $N = 2$ truncation above when $\Theta_+ = \tilde{\Theta}_+ = 0$. Hence, despite having the right electric charges, the supersymmetric $N = 8$ solution we found does not fit into the standard "STU" $N = 2$ truncation. In the next section we discuss the supersymmetry of this solution, and how it fits into a larger $N = 2$ truncation.

II. SUPERSYMMETRY IN $N = 8$ AND $N = 2$

The solution (1) and (2) is a Calabi-Yau four-fold flux background and hence preserves at least four supercharges [17]. We analyze the supersymmetry in detail and then discuss whether the solution and its supercharges fit inside the largest $N = 2$ truncation of the $N = 8$ theory.

Clearly, the hyperkähler background breaks half of the supersymmetry, as it admits only a covariant spinor of (say) positive chirality. This corresponds to the projection $\Gamma^{1234}\eta = -\eta$, where η is a spinor on the internal eight-dimensional manifold. Furthermore, the flux F_4 breaks more supersymmetry. Its electric component (corresponding to an M2-brane charge along the 9, 10 directions) breaks another half of supersymmetry, by the projection $\Gamma^{12345678}\eta = \eta$.

To understand how the magnetic components of F_4 affect the supersymmetry, it is best to choose an appropriate vierbein e^i, $i = 1, \ldots, 4$, on the hyperkähler space M_4, such that (3) is fulfilled and we can identify the self-dual two-forms of (2) as

$$\begin{aligned} \Theta_+ &= \theta_+(e^1 \wedge e^3 + e^4 \wedge e^2), \\ \tilde{\Theta}_+ &= \theta_+(e^1 \wedge e^4 + e^2 \wedge e^3). \end{aligned} \tag{8}$$

The supersymmetry conditions $\not{F}\eta = 0$ and $\not{F}_m\eta = 0$ [17] contain an additional projector, which further halves the amount of supersymmetry. The first condition gives

$$\begin{aligned} &\frac{1}{4}[(\Theta_+)_{ij}\Gamma^{ij58} + (\tilde{\Theta}_+)_{ij}\Gamma^{ij68}](1 - \Gamma^{5678})(1 - \Gamma^{1234})\eta \\ &\quad - \frac{1}{4}(\Theta_-)_{ij}\Gamma^{ij58}(1 + \Gamma^{5678})(1 + \Gamma^{1234})\eta = 0, \end{aligned} \tag{9}$$

where we have inserted the projectors $\frac{1}{2}(1 \pm \Gamma^{1234})$ by making use of the (anti-)self-duality of $\Theta_\mp$.

The term containing the anti-self-dual flux Θ_- vanishes on the Killing spinors annihilated by the two earlier projectors $\frac{1}{2}(1 + \Gamma^{1234})$ and $\frac{1}{2}(1 - \Gamma^{12345678})$, and this agrees with the known structure of BPS three-charge solutions, in which turning on an anti-self-dual field strength on the base does not affect supersymmetry.

For arbitrary self-dual forms Θ_+, $\tilde{\Theta}_+$, the first line is not zero and supersymmetry is broken. However, for the specific choice (8) this term contains a new projector,

$$0 = 2\theta_+\Gamma^{1358}(1 + \Gamma^{3456})\eta, \tag{10}$$

which is compatible with the first two. More generally, under the condition (3) we always find such a projector and the solution has four supercharges.

It is not hard to see that the equations $\not{F}_m\eta = 0$ do not impose any extra conditions on the remaining Killing spinors, essentially because the flux pieces that are self-dual on the hyperkähler manifold always combine into the projector $\frac{1}{2}(1 + \Gamma^{3456})$, while the anti-self-dual components give either $\frac{1}{2}(1 + \Gamma^{1234})$ or $\frac{1}{2}(1 + \Gamma^{5678})$, depending on the index m. Therefore, the solution is 1/8 BPS. Its four Killing spinors are annihilated by the projectors

$$\frac{1}{2}(1 + \Gamma_{1234}), \quad \frac{1}{2}(1 + \Gamma_{3456}) \quad \text{and} \quad \frac{1}{2}(1 + \Gamma_{5678}). \tag{11}$$

The 1/8 BPS solution we gave in (1) and (2) has not been found in the literature. Moreover, its magnetic field strength (2) has both self-dual and anti-self-dual components on the hyperkähler space. This is surprising since all $1/2$ BPS solutions in $N = 2$ supergravity in five dimensions have only anti-self-dual fluxes on the hyperkähler space, as shown in Refs. [5,6]. This indicates that our solution cannot be a $1/2$ BPS solution of $N = 2$ supergravity. In the following we want to discuss what happens to the $1/8$ BPS solution (1) and (2) when mapped to the maximal $N = 2$ truncation of $N = 8$ supergravity.

In order to find a supergravity with eight supercharges in five dimensions, we have to perform a truncation of $N = 8$ supergravity. The field content of these truncated theories (also called "magical supergravities") has been discussed for instance in Refs. [37,38]. The $N = 2$ truncation with the maximal field content (and only vector multiplets) is the magical supergravity related to the Jordan algebra over the quaternions and it admits the global symmetry group $SU^*(6)$. It has the same bosonic field content as five-dimensional $N = 6$ supergravity. As we show in a more detailed work [18], the projection to this $N = 2$ supergravity in five dimensions corresponds to fixing a complex structure I on T^6 and projecting out some representations of the related $\mathrm{SL}(3, \mathbb{C})$. The surviving vector fields of the $N = 2$ projection contain all gauge fields coming from the 11-dimensional three-form potential with two legs on T^6 that are $(1, 1)$ with respect to I. Note that I does not have to be related to the complex structure under which $\mathrm{d}z$ and $\mathrm{d}w$ are holomorphic, as long as the metric given in (1) respects it. If we choose a complex structure I on T^6 such that $\mathrm{d}z^1 = \mathrm{d}x^8 + i\mathrm{d}x^5$, $\mathrm{d}z^2 = \mathrm{d}x^6 + i\mathrm{d}x^7$ and $\mathrm{d}z^3 = \mathrm{d}x^9 + i\mathrm{d}x^{10}$ are holomorphic one-forms under I, then the flux given in (2) is $(1, 1)$ on T^6, and we see that our solution indeed gives a solution to $N = 2$ supergravity.

Now let us understand the amount of supersymmetry of the solution in $N = 2$ supergravity. The complex structure above is different from the complex structure chosen in (6), and under the new complex structure the flux F_4 (5) has a piece that is $(3, 1) \oplus (1, 3)$ and therefore the configuration is not supersymmetric in $N = 2$ supergravity. More precisely, the projection to $N = 2$ breaks the $N = 8$ R-symmetry group $USp(8)$ to $USp(6) \times SU(2)$, where the latter factor is the R-symmetry of the $N = 2$ theory. The action of $USp(6)$ on the spinors defines the projection to $N = 2$. The generator $C \equiv \frac{1}{2}(\Gamma^{85} - \Gamma^{67})$ commutes with the complex structure I, the Cartan generator of $SU(2)$, and hence is a generator of $USp(6)$. In particular, the requirement $C\eta = 0$ implies

$$\frac{1}{2}(1 - \Gamma^{5678})\eta = 0. \tag{12}$$

This projects out all four Killing spinors of the $1/8$ BPS solution, cf. (11). Hence, when we projected to the $N = 2$ $SU^*(6)$ supergravity, we projected out all supercharges which remain unbroken in the solution (1) and (2). Therefore, the solution is nonBPS in $N = 2$ supergravity.

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